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MI-113 - Draft

Evaluation of Beam Coupling due to the Dipole Current Loop

1-Introduction and Preliminary

The conductor arrangement in each Main Injector dipole is such that half of the the current rotates around the beam pipe by almost a turn. This is the systematic source of coupling. In this note, the coupling strength of the induced longitudinal field is estimated and compared to the strength of the skew quadrupole correctors.

In the Guignard approach to weak linear coupling (1,2,3,4), the driving terms of the coupling resonances are the complex coefficients:

$$C = \frac{1}{2\pi R} \int \sqrt{\beta_x \beta_y}\, \frac{R^2}{|B\rho|} \left[\frac{\partial B_x}{\partial \theta} + \frac{1}{2} B_s \left[\frac{\alpha_x}{\beta_x} - \frac{\alpha_z}{\beta_z} \right] - \frac{1}{2} B_s \left(\frac{1}{\beta_x} + \frac{1}{\beta_z} \right) \right] \cdot \exp\left[i \int \left(\frac{R}{\beta_x} - \frac{R}{\beta_z} \right) d\theta' - (Q_x - Q_y)(\theta - \theta') \right] d\theta$$

where:

Qx: horizontal tune
Qy: vertical tune
Bx, By, Bs: the components of the magnetic field.
$\alpha_x, \beta_x, \ldots$: Twiss parameters

Full compensation is achieved when: $C = 0$

It is clear from the above equation that solenoidal fields can be compensated by skew quadrupoles. In the Main Injector case the longitudinal field is distributed along the dipoles, and we may expect that these contributions are not in phase. On the other hand, the skew quadrupole correctors are localized families, and therefore more effective in contributing to the coupling coefficient.

2-Comparison between dipole longitudinal field and skew quadrupoles

In this section we assume that the two coupling fields are uniformly distributed around the ring (2,4). The betatron motion is also simplified to a smooth sinusoidal oscillation (2,4). From previous section arguments, we believe this is a worst scenario. In any case, a more realistic approach will have to incorporate the random tilts of the quadrupole magnets.

Following reference (2,4), the equations of transverse motion in the presence of coupling are:

$$x'' + \left(\frac{Q_x}{R}\right)^2 x = -kz - bz'$$

$$z'' + \left(\frac{Q_z}{R}\right) z = -kx + bx'$$

2.1-Contribution of the dipoles:

b is the axial field contribution:

$$b = \frac{1}{B\rho} B_s$$

In the Main Injector case there are (216 + 128) bending dipoles, with one half-loop (half the current X one turn) per dipole.

Each dipole contributes to: $\int B\,dl = \frac{1}{2}\mu_0 I$ (Ampere's law)

$= 0.0119$ tesla . m @ 9500 amp

Spreading this field around the ring:

$$b = \frac{(216+128) \times \int B\,dl}{\mathcal{C} \times (B\rho)}$$

$$b = 1.2 \times 10^{-6}\ m^{-1}$$

($\mathcal{C}$: circumference)

The coupling coefficient is then (4): Cb = $b \times R$

with Q being the average tune (near the difference resonance), and R is the average ring radius.

2.2-Contribution of the skew quadrupole correctors:

k is the skew quadrupole contribution:

$$k = \frac{1}{2B\rho}\left[\frac{\partial B_x}{\partial x} - \frac{\partial B_y}{\partial y}\right]$$

From Alex Bocagz latest proposal, (MI-0085) there are 4 groups of 4 skew quadrupoles.

Each skew quadrupole has a focusing power:

$$\frac{1}{f} = k \times \ell = 3.5 \times 10^{-4} \quad m^{-1}$$

Spreading this field around the ring:

$$k = 16 \times \frac{1}{f} \times \frac{1}{C}$$

$$k = 1.7 \times 10^{-6} \quad m^{-2}$$

The coupling coefficient is then (4): $Ck = \dfrac{k R^2}{\varphi}$

2.3-Comparison between the two coupling coefficient

Compared to the correcting skew quadrupoles, the relative strength of the current loop is:

$$\frac{C_b}{C_k} = \frac{b\,\varphi}{k\,R} \simeq 0.03$$

This is a negligible effect and can easily be corrected.

3-Conclusions and future directions

The estimates in section 2 show that the coupling due to the dipoles is small compared to the strength of the skew quadrupoles, and the equation of section 1 proves that a compensation scheme is feasible. A more accurate modelling with MAD is an easy task and is planned. In addition to computing the settings of the skew quadrupoles, this modelling will look at the effect on the vertical dispersion.

REFERENCES

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